Journal of Artificial Intelligence and Control Systems

CoSci Press

Research Article

Distributed Fuzzy Adaptive Containment Control for P-Normal Multiagent Systems with Unknown Time-Varying Parameters and Sensor Faults

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Article History:

Received: 2 April 2026

Revised: 8 May 2026

Accepted: 15 June 2026

Published: In Progress

Abstract: A fuzzy adaptive containment control issue is researched for a class of nonlinear p-normal multiagent systems with sensor faults, disturbances and unknown time-varying parameters. Unlike the structural features of existing results, the dynamic of each agent is a power exponential function. The adding-a-power-integrator technique is employed to handle the positive odd-integer powers intrinsic to p-normal dynamics. In order to simplify the calculation steps, a novel compensation controller is proposed to resolve sensor faults by using the Nussbaum gain technology. Based on approximation characteristics of fuzzy logic systems, a nonlinear disturbance observer is designed to estimate unknown composite disturbances consisted of external disturbances and approximation errors. In view of the Lyapunov function method, all closed-loop signals are proved to be bounded. Finally, a simulation example is given to verify the effectiveness of the proposed control protocol.

Keywords: adaptive containment control; disturbance observer; p-normal multiagent systems; sensor faults

1. Introduction

The containment control of multiagent systems (MASs) has attracted wide attention for its applications in practical systems such as mobile robots, multiple ships and multiple vehicles [1–8]. Since the necessary and sufficient conditions for the containment control problem of MASs have been proposed in [9], many containment control results have been gained in [10–13]. In [14], the containment control problem of MASs was researched via utilizing a distributed dynamic state feedback control strategy. A novel distributed error term was designed for containment control of MASs in [15]. It is worth noting that the above control strategies only focus on the MASs where the power of the control items is one, rather than nonlinear p-normal MASs. Compared with the MASs of power one, nonlinear p-normal MASs are more general and complicated in [16–18]. Therefore, the containment control problem of nonlinear p-normal MASs is quite challenging.

Sensor fault is a common phenomenon in practical applications, which has a negative impact on system performance if not adequately addressed. To solve the problem of sensor faults, many efficacious fault-tolerant control strategies were proposed in [19–21], but these approaches do not account for the simultaneous presence of disturbances or approximation errors. In [22], the problem of sensor faults was solved by an adaptive neural network control technique. The authors incorporated sensor faults into the neural network during the derivation process. In order to reduce the containment error, state observers were designed to estimate sensor faults in [23], but the process is relatively complex. In

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addition, the above papers do not consider unknown composite terms composed of external disturbances and approximation errors, which would reduce the robustness of the system. For nonlinear p-normal MASs, it is still a challenge to simultaneously handle sensor faults and disturbances.

Unknown time-varying parameters and disturbances are adverse factors to the control performance of MASs, which often present when the actual system operates. To avert unknown time-varying parameters and disturbances from damaging system performance, a large number of effective control methods have been developed in [24–26]. In [27], for nonlinear MASs with disturbances, a disturbance observer was designed to estimate the disturbances. Unknown time-varying parameters were processed in [28] by establishing distributed adaptive laws. Li et al. [29] proposed a smooth function to overcome the difficulty of unknown time-varying parameters. The above control schemes do not address the impact of disturbances and unknown time-varying parameters under nonlinear p-normal MASs, and the proposed control methods cannot be directly applied to nonlinear p-normal MASs with sensor faults. Compared to dealing with external disturbances, sensor faults, or unknown time-varying parameters separately, it is of great significance to simultaneously handle them and achieve the adaptive containment control for time-varying p-normal MASs.

Based on the above discussions, this paper designs a fuzzy adaptive containment control scheme for the nonlinear p-normal MASs with sensor faults, disturbances and unknown time-varying parameters. The main contributions are as follows.

(1) A novel adaptive compensation controller integrating the power integrator technique is proposed using the Nussbaum gain technique to compensate for the impact of sensor faults in nonlinear p-normal MASs. Compared to designing a state observer to handle the disturbances in [23], the method proposed in this paper is more concise due to the reduction of design parameters in the derivation process.

(2) The use of fuzzy logic systems (FLSs) effectively addresses the unknown time-varying parameters. The external disturbances and approximation errors generated by FLSs are resolved by designing a disturbance observer. Unlike the observer in [27], the observer proposed in this paper is more general due to the presence of power exponent terms in the design process.

2. Preliminaries

2.1. Graph Theory

In this paper, the information exchange between N followers and M leaders is indicated by a directed graph $\mathrm{G} \triangleq (\mathrm{T}_V, \mathrm{T}_E)$. The graph G consists of a node set $\mathrm{T}_V = (1,\ldots,N,N+1,\ldots,N+M)$ and a set of edges $\mathrm{T}_E \subseteq \mathrm{T}_V \times \mathrm{T}_V$. The adjacency matrix is represented as $\mathrm{T}_A = [a_{i,j}] \in \mathbb{R}^{(N+M)\times(N+M)}$. $\mathrm{T}_D = \mathrm{diag}\{d_1,\ldots,d_N,d_{N+1},\ldots,d_{N+M}\}$ is the degree matrix, where $d_i = \sum_{j=1,j\neq i}^{N+M} a_{i,j}$, The Laplacian matrix is represented as $\mathrm{T}_L = [L_{i,j}] \in \mathbb{R}^{(N+M)\times(N+M)} = \mathrm{T}_D - \mathrm{T}_A$, where $L_{i,j} = -a_{i,j}$, if $i \neq j$, and $L_{i,j} = \sum_{j\in \mathrm{V},j\neq i} a_{i,j}$. The Laplacian matrix is

$$\mathrm{T}_L = \begin{bmatrix} \mathrm{T}_{L_1} & \mathrm{T}_{L_2} \\ 0_{M\times N} & 0_{M\times M} \end{bmatrix}$$

where $\mathrm{T}_{L_1} \in \mathbb{R}^{N\times N}$ and $\mathrm{T}_{L_2} \in \mathbb{R}^{N\times M}$. Each follower has at least one route to obtain the information from one of leaders.

2.2. Problem Formulation

Consider the following nonlinear p-normal MASs:

$$\begin{cases} \dot{x}_{i,q} = \tilde{\mathrm{n}}_{i,q}(\bar{x}_{i,q}) x_{i,q+1}^{p_{i,q}} + \psi_{i,q}^T(t)\Xi_{i,q}(\bar{x}_{i,q}), (1 \le q \le n-1) \\ \dot{x}_{i,n} = \tilde{\mathrm{n}}_{i,n}(\bar{x}_{i,n}) u_{i,n}^{p_{i,n}} + \psi_{i,n}^T(t)\Xi_{i,n}(\bar{x}_{i,n}) + d_{i,n}(t) \\ y_i = x_{i,1} \end{cases} \tag{2}$$

where $\bar{x}_{i,q} = [x_{i,1}, x_{i,2},\ldots,x_{i,q}]^T \in \mathbb{R}^q$ and $\bar{x}_{i,n} = [x_{i,1}, x_{i,2},\ldots,x_{i,n}]^T \in \mathbb{R}^n$ are the state vectors. $u_{i,n}$ represents the control input signal. $\tilde{\mathrm{n}}_{i,q}(\bar{x}_{i,q}) \in R$ and $\tilde{\mathrm{n}}_{i,q}(\bar{x}_{i,q}) \in R$ are the known virtual control coefficient. The output signal of the system is y_i. $\psi_{i,q}^T(t)$ and $\psi_{i,n}^T(t)$ express unknown time-varying bounded parameters. $\Xi_{i,q}(\bar{x}_{i,q})$ and $\Xi_{i,n}(\bar{x}_{i,n})$ denote known smooth functions. $d_{i,n}(t)$ is the external disturbance, which is the unknown continuous function. It is assumed to be bounded along with its first time derivative, which satisfies $|d_{i,n}(t)| \le \bar{d}_{i,n}$ and $|\dot{d}_{i,n}(t)| \le \bar{d}'_{i,n}$. $\bar{d}_{i,n}$ and $\bar{d}'_{i,n}$ are positive constants.

Sensor faults are presented as:

$$y_i^f(t) = \delta_i y_i(t), 0 < \delta_i < 1 \tag{3}$$

where δ_i is the fault factor.

Remark 1. It is obvious that nonlinear p-normal MASs (1) are more general than the MASs in [30–33] because the parameters

$p_{i,\cdot}$ are designed as the positive odd integers. Further, how to address the containment control problem of nonlinear p-normal MASs (1) with sensor faults is worth exploring.

Define the following coordinate transformations:

$$\begin{cases} z_{i,1} = \sum_{j=1}^{N} a_{i,j}\left(y_i^f - y_j^f\right) + \sum_{j=N+1}^{N+M} a_{i,j}\left(y_i^f - y_{\iota j}\right), \\ z_{i,\bar{s}} = x_{i,\bar{s}} - \alpha_{i,\bar{s}-1}, \qquad \bar{s} = 2,3,\ldots,n. \end{cases} \tag{4}$$

where $\alpha_{i,\bar{s}-1}$ is the virtual variable and $y_{\iota,j}$ is output signal of the leader.

Control objective: A containment control protocol is proposed for nonlinear p-normal MASs with sensor faults, disturbances and unknown time-varying parameters such that the followers' outputs converge to a convex hull established by multiple leaders.

Assumption 1 δ_i in (2) satisfies $0 < \delta_i \le \hat{\delta}_i < 1$ in which $\hat{\delta}_i$ denotes a known positive constant.

Definition 1 [34] There exist Nussbaum-type functions $N(\hat{\rho})$ conforming to

$$\begin{cases} \lim_{\sigma\to\infty} \sup \dfrac{1}{\sigma}\displaystyle\int_0^{\sigma} N(\hat{\rho})dx = +\infty \\ \lim_{\sigma\to\infty} \inf \dfrac{1}{\sigma}\displaystyle\int_0^{\sigma} N(\hat{\rho})dx = -\infty \end{cases}$$

where $N(\hat{\rho}) = \hat{\rho}^2\cos(\hat{\rho})$.

Remark 2. Projection-based adaptive estimation is also commonly used to handle unknown constant gains with known signs. In this paper, although the sign of the sensor fault factor δ_i is known to be $0 < \delta_i < 1$, the virtual control coefficients $\tilde{\text{n}}_{i,q}$ in system (1) are known smooth functions whose signs may vary with changes in the state. Therefore, the sign of $\delta_i\tilde{\text{n}}_{i,q}$ appearing in subsequent derivations is uncertain. If the projection method is used to estimate δ_i (or $1/\delta_i$), the actual control direction will reverse whenever $\tilde{\text{n}}_{i,q}$ crosses zero. Since the projection estimator cannot adapt to such sign changes, the system will lose stability. In contrast, the Nussbaum technique does not require any prior sign information and it can automatically adjust its output to stabilize the system regardless of how the sign changes. This is precisely why the Nussbaum method was selected in this paper.

Lemma 1. [35] Let $V(t) \ge 0$ and $\hat{\Upsilon}(t)$ be the smooth functions in $[0,t]$. For $\bar{a} > 0$, $\bar{b} > 0$ and $\hbar > 0$, one has

$$V(t) \le \bar{a} + e^{-\bar{b}t}\int_0^t \left[\hbar L\left(\hat{\Upsilon}(\bar{k})\right)\dot{\hat{\Upsilon}}(\bar{k}) + 1\right]e^{\bar{b}\bar{k}}d\bar{k}$$

where $V(t)$, $\hat{\Upsilon}(t)$ and $\int_0^t N\left(\hat{\Upsilon}(\bar{k})\right)\dot{\hat{\Upsilon}}(\bar{k})d\bar{k}$ are bounded.

Lemma 2. [36] Let $f(x_i)$ be a continuous function given on a compact set Ω. For any $\bar{\varepsilon} > 0$, FLSs $\grave{o}^{*\mathrm{T}}\Phi(x_i)$ are described as

$$\sup_{x_i\in\Omega}\left| f(x_i) - \grave{o}^T\Phi(x_i)\right| \le \bar{\varepsilon}$$

Lemma 3. [37] For $\mathrm{A}_{i,\mathrm{M}}$ and $\bar{v}_{i,\mathrm{M}} \in R$, the second-order tracking differentiator is given by

$$\begin{cases} \dot{\mathrm{G}}_{i,\mathrm{P}} = \bar{v}_{i,\mathrm{P}} \\ \dot{\bar{v}}_{i,\mathrm{P}} = -\bar{o}_{i,\mathrm{P}}\,\mathrm{sign}\left(\mathrm{G}_{i,\mathrm{P}} - \alpha_{i,\mathrm{P}} + \dfrac{\bar{v}_{i,\mathrm{P}}\left|\bar{v}_{i,\mathrm{P}}\right|}{2\bar{o}_{i,\mathrm{P}}}\right) \end{cases}$$

with $\mathrm{sign}(\bar{\varpi})$ being defined as

$$\mathrm{sign}(\bar{\varpi}) = \begin{cases} 1, & \bar{\varpi} > 0, \\ 0, & \bar{\varpi} = 0, \\ -1, & \bar{\varpi} < 0, \end{cases}$$

where $\bar{o}_{i,\mathrm{P}}$ is a design parameter. There exist $\lim_{\bar{o}_{i,\mathrm{P}}\to\infty}\mathrm{G}_{i,\mathrm{P}} = \alpha_{i,\mathrm{P}}$ and $\lim_{\bar{o}_{i,\mathrm{P}}\to\infty}\bar{v}_{i,\mathrm{P}} = \dot{\alpha}_{i,\mathrm{P}}$.

Lemma 4. [38] For $\hat{x} \in R$, $\hat{y} \in R$ and positive odd integer $\bar{a} \ge 1$, one has

$$\left|\hat{x}^{\bar{a}} - \hat{y}^{\bar{a}}\right| \le \bar{a}\left|\hat{x} - \hat{y}\right|\left(\hat{x}^{\bar{a}-1} - \hat{y}^{\bar{a}-1}\right)$$

The disturbance observer is designed as

$$\begin{cases} \hat{\sigma}_{i,n} = r_{i,n} + q_{i,n} \\ \dot{r}_{i,n} = -\xi_{i,n}r_{i,n} - \xi_{i,n}\left(\text{蜂}_{i,n}^{T}\Phi_{i,n} + {}_{i,n}u_{i,n}^{p_{i,n}} + q_{i,n}\right) + z_{i,n}^{p_i - p_{i,n}+1} \end{cases} \tag{5}$$

where $\xi_{i,n} > 0$ and $q_{i,n} = \xi_{i,n}x_{i,n}$.

3. Main Results

A containment control protocol for nonlinear p-normal MASs with power integrators will be proposed. Define $p_i = \max_{1\le k\le m}\{p_{i,k}\}$ and the ideal FLS parameters norm $\theta_{i,m} = \left\|\grave{o}_{i,m}^{*T\frac{p_i+1}{p_i - p_{i,m}+1}}\right\|$ with $m = 1,2,\ldots,n$.

Step 1 : From (3), the derivative of $z_{i,1}$ is given as

$$\begin{aligned} \dot{z}_{i,1} &= \delta_i\sum_{j=1}^{N}a_{i,j}\left(\text{耨}_{i,1}x_{i,2}^{p_{i,1}} + \psi_{i,1}^T(t)\Xi_{i,1}\right) - \delta_i\sum_{j=1}^{N}a_{i,j}\left({}_{j,1}x_{j,2}^{p_{j,1}} + \psi_{j,1}^T(t)\Xi_{j,1}\right) \\ &\quad + \delta_i\sum_{j=N+1}^{N+M}a_{i,j}\left(\tilde{\text{n}}_{i,1}x_{i,2}^{p_{i,1}} + \psi_{i,1}^T(t)\Xi_{i,1}\right) - \sum_{j=N+1}^{N+M}a_{i,j}\delta_i\dot{y}_{\iota,j} \\ &= \delta_i\bar{\beta}_i\left(\tilde{\text{n}}_{i,1}x_{i,2}^{p_{i,1}} + \psi_{i,1}^T(t)\Xi_{i,1}\right) - \sum_{j=N+1}^{N+M}a_{i,j}\delta_i\dot{y}_{\iota,j} \\ &\quad - \delta_i\sum_{j=1}^{N}a_{i,j}\left(\tilde{\text{n}}_{j,1}x_{j,2}^{p_{j,1}} + \psi_{j,1}^T(t)\Xi_{j,1}\right) \end{aligned}$$

where $\bar{\beta}_i = \sum_{j=1}^{N} a_{i,j} + \sum_{j=N+1}^{N+M} a_{i,j}$, and δ_i is the fault factor.

Choose the Lyapunov function below

$$V_{i,1} = \frac{z_{i,1}^{p_i - p_{i,1}+2}}{p_i - p_{i,1} + 2} + \frac{\tilde{\theta}_{i,1}^2}{2c_{i,1}} \tag{6}$$

where $c_{i,1} > 0$ is a design parameter. $\tilde{\theta}_{i,m} = \theta_{i,m} - \hat{\theta}_{i,m}$. $\hat{\theta}_{i,m}$ means the estimation of $\theta_{i,m}$.

The derivative of (5) is described as

$$\begin{aligned}\dot{V}_{i,1} &= z_{i,1}^{p_i - p_{i,1}+1}\Bigg[\delta_i \bar{\beta}_i \left(\tilde{\mathrm{n}}_{i,1} x_{i,2}^{p_{i,1}} + \psi_{i,1}^T(t)\Xi_{i,1}\right) - \sum_{j=N+1}^{N+M} a_{i,j}\delta_i \dot{y}_{i,j} \\ &\quad -\delta_i \sum_{j=1}^{M} a_{i,j}\left(\tilde{\mathrm{n}}_{j,1} x_{j,2}^{p_{j,1}} + \psi_{j,1}^T(t)\Xi_{j,1}\right)\Bigg] - \frac{1}{c_{i,1}}\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1} \\ &= z_{i,1}^{p_i - p_{i,1}+1}\left[\delta_i \bar{\beta}_i \tilde{\mathrm{n}}_{i,1}\left(x_{i,2}^{p_{i,1}} - \alpha_{i,1}^{p_{i,1}} + \alpha_{i,1}^{p_{i,1}}\right) + \bar{\psi}_{i,1}(\chi_{i,1})\right] \\ &\quad -3\bar{\beta}_i \tilde{\mathrm{n}}_{i,1} z_{i,1}^{p_{i,1}} - \frac{1}{c_{i,1}}\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}\end{aligned} \tag{7}$$

where $\bar{\psi}_{i,1}(\chi_{i,1}) = -\sum_{j=N+1}^{N+M} a_{i,j}\delta_i \dot{y}_{i,j} + \delta_i \bar{\beta}_i \psi_{i,1}^T(t)\Xi_{i,1} + 3\bar{\beta}_i \tilde{\mathrm{n}}_{i,1} z_{i,1}^{p_{i,1}}$ $-\delta_i \sum_{j=1}^{M} a_{i,j}\left(\tilde{\mathrm{n}}_{j,1} x_{j,2}^{p_{j,1}} + \psi_{j,1}^T(t)\Xi_{j,1}\right)$.

For given the arbitrary accuracy $\varepsilon_{i,1} > 0$, there are FLSs such that

$$\bar{\psi}_{i,1}(\chi_{i,1}) = \grave{o}_{i,1}^{*T}\Phi_{i,1} + \nabla_{i,1}, \left|\nabla_{i,1}\right| \le \varepsilon_{i,1} \tag{8}$$

where $\nabla_{i,1}$ defines the approximation error.

It is obtained that

$$\begin{aligned}& z_{i,1}^{p_i - p_{i,1}+1}\bar{\psi}_{i,1}(\chi_{i,1}) \\ &\le z_{i,1}^{p_i - p_{i,1}+1}\left(\grave{o}_{i,1}^{*T}\Phi_{i,1} + \varepsilon_{i,1}\right) \\ &\le \frac{p_i - p_{i,1}+1}{p_i+1} z_{i,1}^{p_i+1} + \frac{p_i - p_{i,1}+1}{p_i+1} z_{i,1}^{p_i+1}\lambda_{i,1}^{\frac{p_i+1}{p_i - p_{i,1}+1}} \|\grave{o}_{i,1}^{*}\|^{\frac{p_i+1}{p_i - p_{i,1}+1}} \\ &\quad \times\left(\Phi_{i,1}^T\Phi_{i,1}\right)^{\frac{p_i+1}{2(p_i - p_{i,1}+1)}} + \frac{p_{i,1}}{p_i+1}\varepsilon_{i,1}^{\frac{p_i+1}{p_{i,1}}} + \frac{p_{i,1}}{p_i+1}\lambda_{i,1}^{-\frac{p_i+1}{p_{i,1}}} \\ &\le z_{i,1}^{p_i+1}\left(\lambda_{i,1}^{\frac{p_i+1}{p_i - p_{i,1}+1}}\theta_{i,1}\left(\Phi_{i,1}^T\Phi_{i,1}\right)^{\frac{p_i+1}{2(p_i - p_{i,1}+1)}} + 1\right)\varepsilon_{i,1}^{\frac{p_i+1}{p_{i,1}}} + \lambda_{i,1}^{-\frac{p_i+1}{p_{i,1}}}\end{aligned} \tag{9}$$

where $\lambda_{i,1} > 0$ is a constant.

Substituting (8) into (6), one has

$$\begin{aligned}\dot{V}_{i,1} &\le z_{i,1}^{p_i - p_{i,1}+1}\delta_i \bar{\beta}_i \tilde{\mathrm{n}}_{i,1}\left(x_{i,2}^{p_{i,1}} - \alpha_{i,1}^{p_{i,1}} + \alpha_{i,1}^{p_{i,1}}\right) - 3\bar{\beta}_i \tilde{\mathrm{n}}_{i,1} z_{i,1}^{p_i+1} \\ &\quad + z_{i,1}^{p_i+1}\left(\lambda_{i,1}^{\frac{p_i+1}{p_i - p_{i,1}+1}}\theta_{i,1}\left(\Phi_{i,1}^T\Phi_{i,1}\right)^{\frac{p_i+1}{2(p_i - p_{i,1}+1)}} + 1\right) + \varepsilon_{i,1}^{\frac{p_i+1}{p_{i,1}}} \\ &\quad + \lambda_{i,1}^{-\frac{p_i+1}{p_{i,1}}} - \frac{1}{c_{i,1}}\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}\end{aligned} \tag{10}$$

The virtual controller is selected as

$$\alpha_{i,1} = -\left(N(\hat{\rho}_{i,1})\right)^{\frac{1}{p_{i,1}}}\vartheta_{i,1} \tag{11}$$

$$\begin{cases} N(\hat{\rho}_{i,1}) = \hat{\rho}_{i,1}^2 \cos(\hat{\rho}_{i,1}) \\ \dot{\hat{\rho}}_{i,1} = -r_{i,1}\vartheta_{i,1}^{p_{i,1}} z_{i,1}^{p_i - p_{i,1}+1} \end{cases}$$

where $\vartheta_{i,1}$ will be designed later and $r_{i,1} > 0$ is a design parameter.

To handle $z_{i,1}^{p_i - p_{i,1}+1}\delta_i \bar{\beta}_i \tilde{\mathrm{n}}_{i,1}\alpha_{i,1}^{p_{i,1}}$, one gets

$$\delta_i \alpha_{i,1}^{p_{i,1}} z_{i,1}^{p_i - p_{i,1}+1} = \frac{1}{r_{i,1}}\left(\delta_i N(\hat{\rho}_{i,1}) + 1\right)\dot{\rho}_{i,1} + \vartheta_{i,1}^{p_{i,1}} z_{i,1}^{p_i - p_{i,1}+1} \tag{12}$$

Then, the inequality (9) is rewritten as

$$\begin{aligned}\dot{V}_{i,1} &\le z_{i,1}^{p_i - p_{i,1}+1}\delta_i \bar{\beta}_i \tilde{\mathrm{n}}_{i,1}\left(x_{i,2}^{p_{i,1}} - \alpha_{i,1}^{p_{i,1}}\right) - 3\bar{\beta}_i \tilde{\mathrm{n}}_{i,1} z_{i,1}^{p_{i,1}} + \varepsilon_{i,1}^{\frac{p_i+1}{p_{i,1}}} + \lambda_{i,1}^{-\frac{p_i+1}{p_{i,1}}} \\ &\quad + \bar{\beta}_i \tilde{\mathrm{n}}_{i,1}\frac{1}{r_{i,1}}\left(\delta_i N(\hat{\rho}_{i,1}) + 1\right)\dot{\hat{\rho}}_{i,1} - \frac{\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}}{c_{i,1}} + \bar{\beta}_i \tilde{\mathrm{n}}_{i,1}\vartheta_{i,1}^{p_{i,1}} z_{i,1}^{p_i - p_{i,1}+1} \\ &\quad + z_{i,1}^{p_i+1}\left(\lambda_{i,1}^{\frac{p_i+1}{p_i - p_{i,1}+1}}\theta_{i,1}\left(\Phi_{i,1}^T\Phi_{i,1}\right)^{\frac{p_i+1}{2(p_i - p_{i,1}+1)}} + 1\right)\end{aligned} \tag{13}$$

where $\vartheta_{i,1}$ is designed as

$$\begin{aligned}\vartheta_{i,1} &= -z_{i,1}\left[\frac{1}{\bar{\beta}_i \tilde{\mathrm{n}}_{i,1}}\left(\kappa_{i,1} + \lambda_{i,1}^{\frac{p_i+1}{p_i - p_{i,1}+1}}\hat{\theta}_{i,1}\left(\Phi_{i,1}^T\Phi_{i,1}\right)^{\frac{p_i+1}{2(p_i - p_{i,1}+1)}} + 1\right)\right]^{\frac{1}{p_{i,1}}} \\ &= -z_{i,1}\mathrm{H}_{i,1}\end{aligned} \tag{14}$$

$\kappa_{i,1} > 0$ is a design parameter and $\mathrm{H}_{i,1}$ is represented as

$$\mathrm{H}_{i,1} = \left[\frac{1}{\bar{\beta}_i \tilde{\mathrm{n}}_{i,1}}\left(\kappa_{i,1} + \lambda_{i,1}^{\frac{p_i+1}{p_i - p_{i,1}+1}}\hat{\theta}_{i,1}\left(\Phi_{i,1}^T\Phi_{i,1}\right)^{\frac{p_i+1}{2(p_i - p_{i,1}+1)}} + 1\right)\right]^{\frac{1}{p_{i,1}}} \tag{15}$$

The adaptive law is depicted as

$$\dot{\hat{\theta}}_{i,1} = c_{i,1} z_{i,1}^{p_i+1}\lambda_{i,1}^{\frac{p_i+1}{p_i - p_{i,1}+1}}\left(\Phi_{i,1}^T\Phi_{i,1}\right)^{\frac{p_i+1}{2(p_i - p_{i,1}+1)}} - \omega_{i,1}\hat{\theta}_{i,1} \tag{16}$$

where $\omega_{i,1} > 0$ is a design parameter.

According to Young's inequality, one obtains

$$\frac{\omega_{i,1}}{c_{i,1}}\tilde{\theta}_{i,1}\hat{\theta}_{i,1} \le -\frac{\omega_{i,1}}{2c_{i,1}}\tilde{\theta}_{i,1}^2 + \frac{\omega_{i,1}}{2c_{i,1}}\theta_{i,1}^2 \tag{17}$$

Substituting (14)-(16) into (12), yields

$$\begin{aligned}\dot{V}_{i,1} &\le -\kappa_{i,1} z_{i,1}^{p_{i+1}} + z_{i,1}^{p_i - p_{i,1}+1}\delta_i \bar{\beta}_i \tilde{\mathrm{n}}_{i,1}\left(x_{i,2}^{p_{i,1}} - \alpha_{i,1}^{p_{i,1}}\right) \\ &\quad + \bar{\beta}_i \tilde{\mathrm{n}}_{i,1}\frac{1}{r_{i,1}}\left(\delta_i N(\hat{\rho}_{i,1}) + 1\right)\dot{\hat{\rho}}_{i,1} - 3\bar{\beta}_i \tilde{\mathrm{n}}_{i,1} z_{i,1}^{p_i+1} \\ &\quad - \frac{\omega_{i,1}}{2c_{i,1}}\tilde{\theta}_{i,1}^2 + \frac{\omega_{i,1}}{2c_{i,1}}\theta_{i,1}^2 + \varepsilon_{i,1}^{\frac{p_i+1}{p_{i,1}}} + \lambda_{i,1}^{-\frac{p_i+1}{p_{i,1}}}\end{aligned} \tag{18}$$

By applying Lemma 4, the following inequality holds

$$
\begin{aligned}
&\delta_i z_{i,1}^{p_i-p_{i,1}+1}\left(x_{i,2}^{p_{i,1}}-\alpha_{i,1}^{p_{i,1}}\right)\\
&\le \delta_i | z_{i,1}^{p_i-p_{i,1}+1}| \; | x_{i,2}^{p_{i,1}}-\alpha_{i,1}^{p_{i,1}}|\\
&\le \delta_i | z_{i,1}^{p_i-p_{i,1}+1}| p_{i,1}| x_{i,2}-\alpha_{i,1}| \;\; |\left(z_{i,2}+\alpha_{i,1}\right)^{p_{i,1}-1}+\alpha_{i,1}^{p_{i,1}-1}|\\
&\le \delta_i | z_{i,1}^{p_i}| p_{i,1}| z_{i,2}|2^{p_{i,1}-2}(N(\hat{\rho}_{i,1}))^{\frac{p_{i,1}-1}{p_{i,1}}} \mathrm{H}_{i,1}^{p_{i,1}-1}\\
&\quad+\delta_i | z_{i,1}^{p_i}| p_{i,1}| z_{i,2}|\left(N(\hat{\rho}_{i,1})\right)^{\frac{p_{i,1}-1}{p_{i,1}}} \mathrm{H}_{i,1}^{p_{i,1}-1}\\
&\quad+\delta_i | z_{i,1}^{p_i-p_{i,1}+1}| p_{i,1}2^{p_{i,1}-2}| z_{i,2}^{p_{i,1}}|\\
&\le \frac{z_{i,2}^{p_i+1}}{p_i+1}\left(\delta_i p_{i,1}(N(\hat{\rho}_{i,1}))^{\frac{p_{i,1}-1}{p_{i,1}}} \mathrm{H}_{i,1}^{p_{i,1}-1}\right)^{p_i+1}+2\frac{p_i}{p_i+1}z_{i,1}^{p_i+1}\\
&\quad+\frac{p_i-p_{i,1}+1}{p_i+1}z_{i,1}^{p_i+1}+\frac{p_{i,1}}{p_i+1}z_{i,2}^{p_i+1}\left(\delta_i p_{i,1}2^{p_{i,1}-2}\right)^{\frac{p_i+1}{p_{i,1}}}\\
&\quad+\frac{z_{i,2}^{p_i+1}}{p_i+1}\left(\delta_i p_{i,1}2^{p_{i,1}-2}(N(\hat{\rho}_{i,1}))^{\frac{p_{i,1}-1}{p_{i,1}}} \mathrm{H}_{i,1}^{p_{i,1}-1}\right)^{p_i+1}\\
&\le 3z_{i,1}^{p_i+1}+\frac{1}{\bar{\beta}_i\tilde{n}_{i,1}}z_{i,2}^{p_i-p_{i,2}+1}\mathrm{F}_{i,1}
\end{aligned}
\tag{19}
$$

where

$$
\begin{aligned}
\mathrm{F}_{i,1}=\bar{\beta}_i\tilde{n}_{i,1}z_{i,2}^{p_{i,2}}\Bigg[&\left(\delta_i p_{i,1}2^{p_{i,1}-2}\right)^{\frac{p_i+1}{p_{i,1}}}+\left(\delta_i p_{i,1}(N(\hat{\rho}_{i,1}))^{\frac{p_{i,1}-1}{p_{i,1}}} \mathrm{H}_{i,1}^{p_{i,1}-1}\right)^{p_i+1}\\
&+\left(\delta_i p_{i,1}2^{p_{i,1}-2}(N(\hat{\rho}_{i,1}))^{\frac{p_{i,1}-1}{p_{i,1}}} \mathrm{H}_{i,1}^{p_{i,1}-1}\right)^{p_i+1}\Bigg]
\end{aligned}
\tag{20}
$$

Remark 3. All components in $\mathrm{F}_{i,1}$ are bounded. Specifically, $\bar{\beta}$, $p_{i,1}$, $p_{i,2}$, p_i , δ_i are finite constants, $\tilde{n}_{i,1}$ is a continuous and bounded function, $z_{i,2}^{p_{i,2}}$ is an error variable that will be proved bounded, the Nussbaum function $N(\hat{\rho}_{i,1})$ is bounded as proved later, $\mathrm{H}_{i,1}^{p_{i,1}-1}$ is a known bounded function. In summary, $\mathrm{F}_{i,1}$ is bounded.

Substituting (18) into (17) has

$$
\begin{aligned}
\dot{V}_{i,1}\le{}&-\kappa_{i,1}z_{i,1}^{p_{i+1}}+z_{i,2}^{p_i-p_{i,2}+1}\mathrm{F}_{i,1}-\frac{\omega_{i,1}}{2c_{i,1}}\tilde{\theta}_{i,1}^2+\frac{\omega_{i,1}}{2c_{i,1}}\theta_{i,1}^2\\
&+\bar{\beta}_i\tilde{n}_{i,1}\frac{1}{r_{i,1}}\left(\delta_i N(\hat{\rho}_{i,1})+1\right)\dot{\hat{\rho}}_{i,1}+\varepsilon_{i,1}^{\frac{p_i+1}{p_{i,1}}}+\lambda_{i,1}^{-\frac{p_i+1}{p_{i,1}}}
\end{aligned}
\tag{21}
$$

Step 2: From (3), the derivative of $z_{i,2}$ is given as

$$
\begin{aligned}
\dot{z}_{i,2}&=\dot{x}_{i,2}-\dot{\alpha}_{i,1}\\
&=\tilde{n}_{i,2}x_{i,3}^{p_{i,2}}+\psi_{i,2}^T\Xi_{i,2}-\bar{v}_{i,1}-\tau_{i,1}
\end{aligned}
\tag{22}
$$

where $\bar{v}_{i,1}$ is the state of the tracking differentiator and

$$
\tau_{i,1}=\dot{\alpha}_{i,1}-\bar{v}_{i,1}
\tag{23}
$$

$$
\begin{aligned}
\dot{\alpha}_{i,1}={}&\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\left(x_{i,2}^{p_{i,1}}+\psi_{i,1}\right)+\frac{\partial\alpha_{i,1}}{\partial y_{i,j}}y_{i,j}^{(2)}\\
&+\frac{\partial\alpha_{i,1}}{\partial x_{j,1}}\left(x_{j,2}^{p_{j,1}}+\psi_{j,1}\right)+\frac{\partial\alpha_{i,1}}{\partial\hat{\theta}_{i,1}}\dot{\hat{\theta}}_{i,1}
\end{aligned}
\tag{24}
$$

Remark 4. The boundedness of the estimation error $\tau_{i,s-1}$ (s=2,3...n) for bounded inputs is a standard property of second-order tracking differentiators. A rigorous proof can be found in [39], where a second-order tracking differentiator is shown to have finite-time convergence and thus bounded estimation error.

Let the Lyapunov function as

$$
V_{i,2}=V_{i,1}+\frac{z_{i,2}^{p_i-p_{i,2}+2}}{p_i-p_{i,2}+2}+\frac{\tilde{\theta}_{i,2}^2}{2c_{i,2}}
\tag{25}
$$

By taking the derivative of the above equation, one can obtain

$$
\begin{aligned}
\dot{V}_{i,2}={}&\dot{V}_{i,1}+z_{i,2}^{p_i-p_{i,2}+1}\left(\tilde{n}_{i,2}x_{i,3}^{p_{i,2}}+\psi_{i,2}^T(t)\Xi_{i,2}-\bar{v}_{i,1}-\tau_{i,1}\right)-\frac{\tilde{\theta}_{i,2}\dot{\hat{\theta}}_{i,2}}{c_{i,2}}\\
\le{}&\dot{V}_{i,1}+z_{i,2}^{p_i-p_{i,2}+1}\Big(\tilde{n}_{i,2}\left(x_{i,3}^{p_{i,2}}-\alpha_{i,2}^{p_{i,2}}+\alpha_{i,2}^{p_{i,2}}\right)-\bar{v}_{i,1}\\
&-\tau_{i,1}+\bar{\psi}_{i,2}(\chi_{i,2})-\mathrm{F}_{i,1}\Big)-\frac{\tilde{\theta}_{i,2}\dot{\hat{\theta}}_{i,2}}{c_{i,2}}
\end{aligned}
\tag{26}
$$

where $\bar{\psi}_{i,2}(\chi_{i,2})=\psi_{i,2}^T(t)\Xi_{i,2}+\mathrm{F}_{i,1}$.

$$
\bar{\psi}_{i,2}(\chi_{i,2})=\grave{o}_{i,2}^{*T}\Phi_{i,2}+\nabla_{i,2},\left|\nabla_{i,2}\right|\le\varepsilon_{i,2}
\tag{27}
$$

where $\nabla_{i,2}$ is the approximation error.

According to the Young's inequality, it follows that

$$
\begin{aligned}
z_{i,2}^{p_i-p_{i,2}+1}\bar{\psi}_{i,2}(\chi_{i,2})&\le z_{i,2}^{p_i-p_{i,2}+1}\left(\grave{o}_{i,2}^{*T}\Phi_{i,2}+\varepsilon_{i,2}\right)\\
&\le z_{i,2}^{p_i+1}\left(\lambda_{i,2}^{\frac{p_i+1}{p_i-p_{i,2}+1}}\theta_{i,2}\left(\Phi_{i,2}^T\Phi_{i,2}\right)^{\frac{p_i+1}{2(p_i-p_{i,2}+1)}}+1\right)\\
&\quad+\varepsilon_{i,2}^{\frac{p_i+1}{p_{i,2}}}+\lambda_{i,2}^{-\frac{p_i+1}{p_{i,2}}}
\end{aligned}
\tag{28}
$$

where $\lambda_{i,2}>0$ is a constant.

Then, the inequality (25) is calculated as

$$
\begin{aligned}
\dot{V}_{i,2}\le{}&\dot{V}_{i,1}+z_{i,2}^{p_i+1}\left(\lambda_{i,2}^{\frac{p_i+1}{p_i-p_{i,2}+1}}\theta_{i,2}\left(\Phi_{i,2}^T\Phi_{i,2}\right)^{\frac{p_i+1}{2(p_i-p_{i,2}+1)}}+1\right)\\
&+z_{i,2}^{p_i-p_{i,2}+1}\tilde{n}_{i,2}\left(x_{i,3}^{p_{i,2}}-\alpha_{i,2}^{p_{i,2}}+\alpha_{i,2}^{p_{i,2}}\right)-z_{i,2}^{p_i-p_{i,2}+1}\bar{v}_{i,1}+\varepsilon_{i,2}^{\frac{p_i+1}{p_{i,2}}}\\
&-z_{i,2}^{p_i-p_{i,2}+1}\mathrm{F}_{i,1}+\lambda_{i,2}^{-\frac{p_i+1}{p_{i,2}}}-\frac{\tilde{\theta}_{i,2}}{c_{i,2}}\dot{\hat{\theta}}_{i,2}-z_{i,2}^{p_i-p_{i,2}+1}\tau_{i,1}
\end{aligned}
\tag{29}
$$

In light of Young's inequality, one gets

$$
z_{i,2}^{p_i-p_{i,2}+1}\tau_{i,1}\le\frac{z_{i,2}^{2(p_i-p_{i,2}+1)}}{2}+\frac{\tau_{i,1}^2}{2}
\tag{30}
$$

The virtual controller is selected as

$$
\begin{aligned}
\alpha_{i,2}^{p_{i,2}} = & -\frac{z_{i,2}^{p_{i,2}}}{\tilde{n}_{i,2}}\left(\lambda_{i,2}^{\frac{p_i+1}{p_i-p_{i,2}+1}}\hat{\theta}_{i,2}\left(\Phi_{i,2}^T\Phi_{i,2}\right)^{\frac{p_i+1}{2(p_i-p_{i,2}+1)}}+\kappa_{i,2}+1\right) \\
& +\frac{\bar{v}_{i,1}}{\text{耨}_{i,2}}+\frac{z_{i,2}^{p_i-p_{i,2}+1}}{2\ \ {}_{i,2}}-z_{i,2}^{p_{i,2}}
\end{aligned} \tag{31}
$$

where $\kappa_{i,2}>0$ is a given constant.

The adaptive law is depicted as

$$
\dot{\hat{\theta}}_{i,2}=c_{i,2}z_{i,2}^{p_i+1}\lambda_{i,2}^{\frac{p_i+1}{p_i-p_{i,2}+1}}\left(\Phi_{i,2}^T\Phi_{i,2}\right)^{\frac{p_i+1}{2(p_i-p_{i,2}+1)}}-\omega_{i,2}\hat{\theta}_{i,2} \tag{32}
$$

where $\omega_{i,2}>0$ is a design parameter.

According to Young's inequality, one obtains

$$
\frac{\omega_{i,2}}{c_{i,2}}\tilde{\theta}_{i,2}\hat{\theta}_{i,2}\le -\frac{\omega_{i,2}}{2c_{i,2}}\tilde{\theta}_{i,2}^2+\frac{\omega_{i,2}}{2c_{i,2}}\theta_{i,2}^2 \tag{33}
$$

On the basis of Lemma 4, the remaining item $z_{i,2}^{p_i-p_{i,2}+1}\left(x_{i,3}^{p_{i,2}}-\alpha_{i,2}^{p_{i,2}}\right)$ is processed as

$$
\begin{aligned}
& z_{i,2}^{p_i-p_{i,2}+1}\left(x_{i,3}^{p_{i,2}}-\alpha_{i,2}^{p_{i,2}}\right)\le\left|z_{i,2}^{p_i-p_{i,2}+1}\left(x_{i,3}^{p_{i,2}}-\alpha_{i,2}^{p_{i,2}}\right)\right| \\
& \le\left(x_{i,3}^{p_{i,2}-1}+\alpha_{i,2}^{p_{i,2}-1}\right)p_{i,2}\left|z_{i,2}\right|^{p_i-p_{i,2}+1}\left|x_{i,3}-\alpha_{i,2}\right| \\
& \le p_{i,2}\left|z_{i,2}\right|^{p_i-p_{i,2}+1}\left|z_{i,3}\right|\left(2^{p_{i,2}-1}+1\right)\alpha_{i,2}^{p_{i,2}-1} \\
& \quad +p_{i,2}\left|z_{i,2}\right|^{p_i-p_{i,2}+1}\left|z_{i,3}\right|^{p_{i,2}}2^{p_{i,2}-1} \\
& \le z_{i,3}^{p_i-p_{i,3}+1}\left(h_{i,2}z_{i,3}^{p_{i,3}}+\bar{h}_{i,2}z_{i,3}^{\bar{p}_{i,3}}\right)+z_{i,2}^{p_{i+1}}
\end{aligned} \tag{34}
$$

where

$$
\begin{cases}
h_{i,m}=\dfrac{p_{i,m}}{p_i+1}\left(\dfrac{2(p_i-p_{i,m}+1)}{p_i+1}\right)^{\frac{p_i-p_{i,m}+1}{p_{i,m}}}\left(p_{i,m}2^{p_{i,m}-1}\right)^{\frac{p_i+1}{p_{i,m}}} \\
\bar{h}_{i,m}=\dfrac{p_{i,m}}{p_i+1}\left(\dfrac{2(p_i-p_{i,m}+1)}{p_i+1}\right)^{\frac{p_i-p_{i,m}+1}{p_{i,m}}}\left(p_{i,m}(2^{p_{i,m}-1}+1)\alpha_{i,m}^{p_{i,m}-1}\right)^{\frac{p_i+1}{p_{i,m}}} \\
\bar{p}_{i,m+1}=\dfrac{p_i+1}{p_{i,m}}-(p_i-p_{i,m+1}+1)
\end{cases}
$$

Substituting (29)-(33) into (28), one yields

$$
\begin{aligned}
\dot{V}_{i,2}\le & \dot{V}_{i,1}-\kappa_{i,2}z_{i,2}^{p_{i+1}}+\tilde{n}_{i,2}z_{i,3}^{p_i-p_{i,3}+1}\left(h_{i,2}z_{i,3}^{p_{i,3}}+\bar{h}_{i,2}z_{i,3}^{\bar{p}_{i,3}}\right)+\frac{\tau_{i,1}^2}{2} \\
& -z_{i,l}^{p_i-p_{i,2}+1}\mathrm{F}_{i,1}-\frac{\omega_{i,2}}{2c_{i,2}}\tilde{\theta}_{i,2}^2+\frac{\omega_{i,2}}{2c_{i,2}}\theta_{i,2}^2+\varepsilon_{i,2}^{\frac{p_i+1}{p_{i,2}}}+\lambda_{i,2}^{-\frac{p_i+1}{p_{i,2}}}
\end{aligned} \tag{35}
$$

Step s $(s=3,\ldots,n-1)$ **:** From (3) and Lemma 2, the derivative of $z_{i,s}$ is given as

$$
\begin{aligned}
\dot{z}_{i,s} & =\dot{x}_{i,s}-\dot{\alpha}_{i,s-1} \\
& =\tilde{n}_{i,s}x_{i,s+1}^{p_{i,s}}+\psi_{i,s}^T(t)\Xi_{i,s}-\bar{v}_{i,s-1}-\tau_{i,s-1}
\end{aligned} \tag{36}
$$

where

$$
\tau_{i,s-1}=\dot{\alpha}_{i,s-1}-\bar{v}_{i,s-1} \tag{37}
$$

$$
\begin{aligned}
\dot{\alpha}_{i,s-1}= & \sum_{k=1}^{s-1}\frac{\partial\alpha_{i,s-1}}{\partial x_{i,k}}\left(x_{i,k+1}^{p_{i,k}}+\psi_{i,k}\right)+\sum_{k=1}^{s-1}\frac{\partial\alpha_{i,s-1}}{\partial y_{i,j}^{(k)}}y_{i,j}^{(k+1)} \\
& +\sum_{k=1}^{s-1}\sum_{j\in N_i}\frac{\partial\alpha_{i,s-1}}{\partial x_{j,k}}\left(x_{j,k+1}^{p_{j,k}}+\psi_{j,k}\right)+\sum_{k=1}^{s-1}\frac{\partial\alpha_{i,s-1}}{\partial\hat{\theta}_{i,k}}\dot{\hat{\theta}}_{i,k}
\end{aligned} \tag{38}
$$

Let the Lyapunov function as

$$
V_{i,s}=\frac{z_{i,s}^{p_i-p_{i,s}+2}}{p_i-p_{i,s}+2}+\frac{\tilde{\theta}_{i,s}^2}{2c_{i,s}}+V_{i,s-1} \tag{39}
$$

The time derivative of $V_{i,s}$ is

$$
\begin{aligned}
\dot{V}_{i,s}= & \dot{V}_{i,s-1}+z_{i,s}^{p_i-p_{i,s}+1}\left(\tilde{n}_{i,s}x_{i,s+1}^{p_{i,s}}+\psi_{i,s}^T(t)\Xi_{i,s}-\bar{v}_{i,s-1}-\tau_{i,s-1}\right) \\
& -\frac{1}{c_{i,s}}\tilde{\theta}_{i,s}\dot{\hat{\theta}}_{i,s} \\
\le & \dot{V}_{i,s-1}+z_{i,s}^{p_i-p_{i,s}+1}\tilde{n}_{i,s}\left(x_{i,s+1}^{p_{i,s}}-\alpha_{i,s}^{p_{i,s}}+\alpha_{i,s}^{p_{i,s}}\right)-\frac{\tilde{\theta}_{i,s}}{c_{i,s}}\dot{\hat{\theta}}_{i,s} \\
& +z_{i,s}^{p_i-p_{i,s}+1}\bar{\psi}_{i,s}(\chi_{i,s})-z_{i,s}^{p_i-p_{i,s}+1}\bar{v}_{i,s-1}-z_{i,s}^{p_i-p_{i,s}+1}\tau_{i,s-1}
\end{aligned} \tag{40}
$$

where $\bar{\psi}_{i,s}(\chi_{i,s})=\psi_{i,s}^T(t)\Xi_{i,s}(\bar{x}_{i,s})$. According to Lemma 2, one gets

$$
\bar{\psi}_{i,s}(\chi_{i,s})=\eth_{i,s}^{*T}\Phi_{i,s}(\chi_{i,s})+\nabla_{i,s},\left|\nabla_{i,s}\right|\le\varepsilon_{i,s} \tag{41}
$$

Similar to inequality (27), it can obtain

$$
\begin{aligned}
z_{i,s}^{p_i-p_{i,s}+1}\bar{\psi}_{i,s}(\chi_{i,s}) & \le z_{i,s}^{p_i-p_{i,s}+1}\left(\eth_{i,s}^{*T}\Phi_{i,s}(\chi_{i,s})+\varepsilon_{i,s}\right) \\
& \le z_{i,s}^{p_i+1}\left(\lambda_{i,s}^{\frac{p_i+1}{p_i-p_{i,s}+1}}\theta_{i,s}\left(\Phi_{i,s}^T(\chi_{i,s})\Phi_{i,s}(\chi_{i,s})\right)^{\frac{p_i+1}{2(p_i-p_{i,s}+1)}}\right. \\
& \quad \left.+1\right)+\varepsilon_{i,s}^{\frac{p_i+1}{p_{i,s}}}+\lambda_{i,s}^{-\frac{p_i+1}{p_{i,s}}}
\end{aligned} \tag{42}
$$

where $\lambda_{i,s}$ stands for a positive constant.

The virtual controller can be selected as

$$
\begin{aligned}
\alpha_{i,s}^{p_{i,s}}= & -\frac{z_{i,s}^{p_{i,s}}}{\tilde{n}_{i,s}}\left(\lambda_{i,s}^{\frac{p_i+1}{p_i-p_{i,s}+1}}\hat{\theta}_{i,s}\left(\Phi_{i,s}^T(\chi_{i,s})\Phi_{i,s}(\chi_{i,s})\right)^{\frac{p_i+1}{2(p_i-p_{i,s}+1)}}+1+\kappa_{i,s}\right) \\
& +\frac{\bar{v}_{i,s-1}}{\text{耨}_{i,s}}-\frac{z_{i,s}^{p_i-p_{i,s}+1}}{2\ \ {}_{i,s}}-z_{i,s}^{p_{i,s}}-\frac{\text{耨}_{i,s-1}}{\text{耨}_{i,s}}h_{i,s-1}z_{i,s}^{p_{i,s}}-\frac{{}_{i,s-1}}{{}_{i,s}}\bar{h}_{i,s-1}z_{i,s}^{\bar{p}_{i,s}}
\end{aligned} \tag{43}
$$

where $\kappa_{i,s}$ indicates a positive parameter.

The adaptive law is depicted by

$$
\dot{\hat{\theta}}_{i,s}=c_{i,s}z_{i,s}^{p_i+1}\lambda_{i,s}^{\frac{p_i+1}{p_i-p_{i,s}+1}}\left(\Phi_{i,s}^T(\chi_{i,s})\Phi_{i,s}(\chi_{i,s})\right)^{\frac{p_i+1}{2(p_i-p_{i,s}+1)}}-\omega_{i,s}\hat{\theta}_{i,s} \tag{44}
$$

where $\omega_{i,s}>0$ is a design parameter.

According to Young's inequality, one gets

$$
\frac{\omega_{i,s}}{c_{i,s}}\tilde{\theta}_{i,s}\hat{\theta}_{i,s}\le -\frac{\omega_{i,s}}{2c_{i,s}}\tilde{\theta}_{i,s}^2+\frac{\omega_{i,s}}{2c_{i,s}}\theta_{i,s}^2 \tag{45}
$$

Similar to the calculation (33) in Step 2, one obtains

$$
\begin{aligned}
& z_{i,s}^{\,p_i-p_{i,s}+1}\left(x_{i,s+1}^{p_{i,s}}-\alpha_{i,s}^{p_{i,s}}\right) \\
& \le \left| z_{i,s}^{p_i-p_{i,s}+1}\left(x_{i,s+1}^{p_{i,s}}-\alpha_{i,s}^{p_{i,s}}\right)\right| \\
& \le \left(x_{i,s+1}^{p_{i,s}-1}+\alpha_{i,s}^{p_{i,s}-1}\right) p_{i,s}\left|z_{i,s}\right|^{p_i-p_{i,s}+1}\left|x_{i,s+1}-\alpha_{i,s}\right| \\
& \le p_{i,s} z_{i,s}^{\,p_i-p_{i,s}+1} z_{i,s+1}\left(2^{p_{i,s}-1}+1\right)\alpha_{i,s}^{p_{i,s}-1} \\
& \quad + p_{i,s} z_{i,s}^{\,p_i-p_{i,s}+1} z_{i,s+1}^{\,p_{i,s}} 2^{p_{i,s}-1} \\
& \le z_{i,s+1}^{p_i-p_{i,s+1}+1}\left(h_{i,s} z_{i,s+1}^{p_{i,s+1}}+\bar{h}_{i,s} z_{i,s+1}^{\bar{p}_{i,s+1}}\right)+z_{i,s}^{p_{i+1}}
\end{aligned} \tag{46}
$$

Substituting (40)-(45) into (39) gives

$$
\begin{aligned}
\dot{V}_{i,s} \le\ & \dot{V}_{i,s-1}+z_{i,s+1}^{p_i-p_{i,s+1}+1}\left(h_{i,s} z_{i,s+1}^{p_{i,s+1}}+\bar{h}_{i,s} z_{i,s+1}^{\bar{p}_{i,s+1}}\right) \\
& -\kappa_{i,s} z_{i,s}^{p_{i+1}}-\frac{\omega_{i,s}}{2c_{i,s}}\tilde{\theta}_{i,s}^2+\frac{\omega_{i,s}}{2c_{i,s}}\theta_{i,s}^2 \\
& +\varepsilon_{i,s}^{\frac{p_i+1}{p_{i,s}}}+\lambda_{i,s}^{-\frac{p_i+1}{p_{i,s}}}+\frac{\tau_{i,s-1}^2}{2}
\end{aligned} \tag{47}
$$

Step n: From (3), $\dot{z}_{i,n}$ is given as

$$
\dot{z}_{i,n}=\tilde{\mathrm{n}}_{i,n} u_{i,n}^{p_{i,n}}+\psi_{i,n}^T \Xi_{i,n}+d_{i,n}-\bar{v}_{i,n-1}-\tau_{i,n-1} \tag{48}
$$

where

$$
\tau_{i,n-1}=\dot{\alpha}_{i,n-1}-\bar{v}_{i,n-1} \tag{49}
$$

$$
\begin{aligned}
\dot{\alpha}_{i,n-1}=\ & \sum_{k=1}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial x_{i,k}}\left(x_{i,k+1}^{p_{i,k}}+\psi_{i,k}\right)+\sum_{k=1}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial y_{i,j}^{(k)}}y_{i,j}^{(k+1)} \\
& +\sum_{k=1}^{n-1}\sum_{j\in N_i}\frac{\partial\alpha_{i,n-1}}{\partial x_{j,k}}\left(x_{j,k+1}^{p_{j,k}}+\psi_{j,k}\right)+\sum_{k=1}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial\hat{\theta}_{i,k}}\dot{\hat{\theta}}_{i,k}
\end{aligned} \tag{50}
$$

Choose the Lyapunov function below

$$
V_{i,n}=V_{i,n-1}+\frac{z_{i,n}^{p_i-p_{i,n}+2}}{p_i-p_{i,n}+2}+\frac{\tilde{\theta}_{i,n}^2}{2c_{i,n}}+\frac{\tilde{\sigma}_{i,n}^2}{2} \tag{51}
$$

By taking the derivative of $V_{i,n}$, one can obtain

$$
\begin{aligned}
\dot{V}_{i,n}=\ & \dot{V}_{i,n-1}+z_{i,n}^{p_i-p_{i,n}+1}\Big(\tilde{\mathrm{n}}_{i,n} u_{i,n}^{p_{i,n}}+\psi_{i,n}^T\Xi_{i,n}+d_{i,n} \\
& -\bar{v}_{i,n-1}-\tau_{i,n-1}\Big)-\frac{\tilde{\theta}_{i,n}}{c_{i,n}}\dot{\hat{\theta}}_{i,n}+\tilde{\sigma}_{i,n}\dot{\tilde{\sigma}}_{i,n}
\end{aligned} \tag{52}
$$

Let $\bar{\psi}_{i,n}(\chi_{i,n})=\psi_{i,n}^T\Xi_{i,n}$ is expressed by FLSs as

$$
\bar{\psi}_{i,n}(\chi_{i,n})=\grave{o}_{i,n}^{*T}\Phi_{i,n}+\nabla_{i,n},\left|\nabla_{i,n}\right|\le\varepsilon_{i,n} \tag{53}
$$

Let

$$
\sigma_{i,n}=\nabla_{i,n}+d_{i,n}(t),\left|\dot{\sigma}_{i,n}\right|\le\varsigma_i \tag{54}
$$

where $\varsigma_i\ge 0$. Substituting (52) and (53) into (51) gives

$$
\begin{aligned}
\dot{V}_{i,n}=\ & \dot{V}_{i,n-1}+z_{i,n}^{p_i-p_{i,n}+1}\Big(\text{耩}_{i,n} u_{i,n}^{p_{i,n}}+{}^{*T}_{i,n}\Phi_{i,n}+\sigma_{i,n} \\
& -\bar{v}_{i,n-1}-\tau_{i,n-1}\Big)-\frac{\tilde{\theta}_{i,n}}{c_{i,n}}\dot{\hat{\theta}}_{i,n}+\tilde{\sigma}_{i,n}\dot{\tilde{\sigma}}_{i,n}
\end{aligned} \tag{55}
$$

Using Young's inequality, one obtains

$$
\begin{aligned}
z_{i,n}^{p_i-p_{i,n}+1}\grave{o}_{i,n}^{*T}\Phi_{i,n}\le\ & z_{i,n}^{p_i+1}\lambda_{i,n}^{\frac{p_i+1}{p_i-p_{i,n}+1}}\theta_{i,n}\left(\Phi_{i,n}^T\Phi_{i,n}\right)^{\frac{p_i+1}{2(p_i-p_{i,n}+1)}} \\
& +\lambda_{i,n}^{-\frac{p_i+1}{p_{i,n}}}
\end{aligned} \tag{56}
$$

where $\lambda_{i,n}>0$ is a constant.

Using the disturbance observer (4) designed above, one gets

$$
\begin{aligned}
-\tilde{\sigma}_{i,n}\dot{\hat{\sigma}}_{i,n} & =-\tilde{\sigma}_{i,n}\left(\dot{r}_{i,n}+\dot{q}_{i,n}\right) \\
& =-\tilde{\sigma}_{i,n}\left(\dot{r}_{i,n}+\dot{\xi}_{i,n}x_{i,n}+\xi_{i,n}\dot{x}_{i,n}\right) \\
& =-\tilde{\sigma}_{i,n}\left[-\xi_{i,n}r_{i,n}-\xi_{i,n}\hat{\grave{o}}_{i,n}^T\Phi_{i,n}-\xi_{i,n}q_{i,n}+z_{i,n}^{p_i-p_{i,n}+1}\right] \\
& \quad -\tilde{\sigma}_{i,n}\xi_{i,n}\left(\grave{o}_{i,n}^{*T}\Phi_{i,n}+\nabla_{i,n}+d_{i,n}(t)\right) \\
& =-\tilde{\sigma}_{i,n}\left[\xi_{i,n}\tilde{\sigma}_{i,n}+\xi_{i,n}\tilde{\grave{o}}_{i,n}^T\Phi_{i,n}+z_{i,n}^{p_i-p_{i,n}+1}\right] \\
& =-\xi_{i,n}\tilde{\sigma}_{i,n}^2-\tilde{\sigma}_{i,n}\xi_{i,n}\tilde{\grave{o}}_{i,n}^T\Phi_{i,n}-\tilde{\sigma}_{i,n}z_{i,n}^{p_i-p_{i,n}+1}
\end{aligned} \tag{57}
$$

Defining $\|\Phi_{i,n}\|\le\ell_{i,n}$, one obtains

$$
\tilde{\sigma}_{i,n}\text{蝌}_{i,n}\Phi_{i,n}\le\frac{1}{2\varpi_{i,n}}\|\tilde{\sigma}_{i,n}\|^2+\frac{\varpi_{i,n}}{2}\|\tilde{}_{i,n}\|^2\ell_{i,n}^2 \tag{58}
$$

where $\ell_{i,n}$ and $\varpi_{i,n}$ are positive design constants.

Adopting Young's inequality, one gets

$$
\tilde{\sigma}_{i,n}\dot{\sigma}_{i,n}\le\frac{1}{2}\tilde{\sigma}_{i,n}^2+\frac{1}{2}\varsigma_i^2 \tag{59}
$$

The controller $u_{i,n}$ is presented as

$$
\begin{aligned}
u_{i,n}=\Bigg[\ & \frac{1}{\tilde{\mathrm{n}}_{i,n}}\Big(-\kappa_{i,n}z_{i,n}^{p_{i,n}}-z_{i,n}^{p_i-p_{i,n}+1}-\hat{\theta}_{i,n}\Phi_{i,n} \\
& -\text{耩}_{i,n-1}h_{i,n-1}z_{i,n}^{p_{i,n}}-{}_{i,n-1}\bar{h}_{i,n-1}z_{i,n}^{\bar{p}_{i,n}}\Big)\Bigg]^{\frac{1}{p_{i,n}}}
\end{aligned} \tag{60}
$$

where $\kappa_{i,n}>0$ is a parameter.

The adaptive law is depicted as

$$
\dot{\hat{\theta}}_{i,n}=-\omega_{i,n}\hat{\theta}_{i,n}+c_{i,n}z_{i,n}^{p_i-p_{i,n}+1}\Phi_{i,n} \tag{61}
$$

where $\omega_{i,n}>0$ is a constant. It is attainable to obtain

$$
\frac{\omega_{i,n}}{c_{i,n}}\tilde{\theta}_{i,n}\hat{\theta}_{i,n}\le\frac{\omega_{i,n}}{2c_{i,n}}\theta_{i,n}^2-\frac{\omega_{i,n}}{2c_{i,n}}\tilde{\theta}_{i,n}^2 \tag{62}
$$

By applying (55)-(61), the equation (54) is rewritten as

$$
\begin{aligned}
\dot{V}_{i,n} \le{}& \dot{V}_{i,n-1} - \kappa_{i,n} z_{i,n}^{p_i+1} - z_{i,n}^{p_i-p_{i,n}+1}\left(h_{i,n-1} z_{i,n}^{p_{i,n}} + \overline{h}_{i,n-1} z_{i,n}^{\overline{p}_{i,n}} \right) \\
&+ z_{i,n}^{p_i-p_{i,n}+1}\tilde{\sigma}_{i,n} + \tilde{\sigma}_{i,n}\dot{\sigma}_{i,n} - \tilde{\sigma}_{i,n}\dot{\hat{\sigma}}_{i,n} \\
&+ \frac{\tau_{i,n-1}^2}{2} - \frac{\omega_{i,n}\tilde{\theta}_{i,n}^2}{2c_{i,n}} + \frac{\omega_{i,n}\theta_{i,n}^2}{2c_{i,n}} + \lambda_{i,n}^{-\frac{p_i+1}{p_{i,n}}} \\
\le{}& \dot{V}_{i,n-1} - \kappa_{i,n} z_{i,n}^{p_i+1} - z_{i,n}^{p_i-p_{i,n}+1}\left(h_{i,n-1} z_{i,n}^{p_{i,n}} + \overline{h}_{i,n-1} z_{i,n}^{\overline{p}_{i,n}} \right) + \frac{\varsigma_i^2}{2} \\
&- \frac{\omega_{i,n}}{2c_{i,n}}\tilde{\theta}_{i,n}^2 + \frac{\omega_{i,n}}{2c_{i,n}}\theta_{i,n}^2 + \lambda_{i,n}^{-\frac{p_i+1}{p_{i,n}}} + \frac{\tilde{\sigma}_{i,n}^2}{2} - \xi_{i,n}\tilde{\sigma}_{i,n}^2 + \frac{\tau_{i,n-1}^2}{2} \\
\le{}& -\sum_{k=1}^{n}\kappa_{i,k} z_{i,k}^{p_i+1} - \frac{1}{2}\sum_{k=1}^{n}\frac{\omega_{i,k}}{c_{i,k}}\tilde{\theta}_{i,k}^2 + \frac{1}{2}\sum_{k=1}^{n}\frac{\omega_{i,k}}{c_{i,k}}\theta_{i,k}^2 + \sum_{k=1}^{n}\lambda_{i,k}^{-\frac{p_i+1}{p_{i,k}}} \\
&+ \sum_{k=1}^{n-1}\varepsilon_{i,k}^{\frac{p_i+1}{p_{i,k}}} + \sum_{k=2}^{n}\frac{\tau_{i,k-1}^2}{2} + \overline{\beta}_i \tilde{n}_{i,1}\frac{1}{r_{i,1}}\left(\delta_i N(\rho_{i,1}) + 1 \right)\dot{\rho}_{i,1} \\
&- \left(\xi_{i,n} - \frac{1}{2} \right)\tilde{\sigma}_{i,n}^2 + \frac{1}{2}\varsigma_i^2 \\
\le{}& -\eta_i V_{i,n} + b_i + \mathrm{W}_{i,1}
\end{aligned}
\tag{63}
$$

where

$$\eta_i = \min\left\{ \kappa_{i,k}, \xi_{i,n} - \frac{1}{2}, \omega_{i,k} \right\}$$

$$b_i = \frac{1}{2}\sum_{k=1}^{n}\frac{\omega_{i,k}}{c_{i,k}}\theta_{i,k}^2 + \sum_{k=1}^{n}\lambda_{i,k}^{-\frac{p_i+1}{p_{i,k}}} + \sum_{k=1}^{n-1}\varepsilon_{i,k}^{\frac{p_i+1}{p_{i,k}}} + \sum_{k=2}^{n}\frac{\tau_{i,k-1}^2}{2} + \frac{1}{2}\varsigma_i^2$$

$$\mathrm{W}_{i,1} = \overline{\beta}_i \tilde{n}_{i,1}\frac{1}{r_{i,1}}\left(\delta_i N(\hat{\rho}_{i,1}) + 1 \right)\dot{\hat{\rho}}_{i,1} \tag{64}$$

4. Stability Analysis

Theorem 1. Considering nonlinear p-normal MASs (1) under Assumption 1, the adaptive laws (15), (31), (43), (60) and the controller (59) are designed, so that the control objective is achieved and all signals of MASs (1) are bounded.

Proof of Theorem 1. The Lyapunov function V is defined as

$$V = \sum_{i=1}^{N} V_{i,m}(t) \tag{65}$$

Based on (62), one has

$$\dot{V} \le -\eta V + b + \mathrm{W} \tag{66}$$

where $\eta = \min\{\eta_i, i = 1, \ldots, N\}$, $b = \sum_{i=1}^{N} b_i$ and $\mathrm{W} = \sum_{i=1}^{N}\mathrm{W}_{i,1}$.

From (65), it can be obtained

$$\frac{d(V(t)e^{\eta t})}{dt} \le e^{\eta t}\mathrm{W} + e^{\eta t} b \tag{67}$$

Thus, one has

$$V(t) \le V(0)e^{-\eta t} + \frac{b}{\eta}\left(1 - e^{-\eta t} \right) + e^{-\eta t}\int_0^t \mathrm{W}\, e^{\eta k} dk \tag{68}$$

According to (63), W can be decomposed as

$$\mathrm{W} = \sum_{i=1}^{N}\frac{\overline{\beta}_i \tilde{n}_{i,1}\delta_i}{r_{i,1}} N(\hat{\rho}_{i,1})\dot{\hat{\rho}}_{i,1} + \sum_{i=1}^{N}\frac{\overline{\beta}_i \tilde{n}_{i,1}}{r_{i,1}}\dot{\hat{\rho}}_{i,1} \tag{69}$$

By Young's inequality, $\sum_{i=1}^{N}\frac{\overline{\beta}_i \tilde{n}_{i,1}}{r_{i,1}}\dot{\hat{\rho}}_{i,1}$ can be absorbed into $-\eta V$ and the constant b. Consequently, there exist new positive constants $-\tilde{\eta}$ and $\tilde{b}$, such that

$$\dot{V} \le -\tilde{\eta} V + \tilde{b} + \sum_{i=1}^{N}\tilde{\hbar}_i N(\hat{\rho}_{i,1})\dot{\hat{\rho}}_{i,1} \tag{70}$$

where $\tilde{\hbar}_i = \frac{\overline{\beta}_i \tilde{n}_{i,1}\delta_i}{r_{i,1}} N(\hat{\rho}_{i,1})\dot{\hat{\rho}}_{i,1}$.

Performing the same integration as from (65) to (67) on (69) yields

$$V(t) \le V(0)e^{-\tilde{\eta} t} + \frac{\tilde{b}}{\tilde{\eta}}\left(1 - e^{-\tilde{\eta} t} \right) + e^{-\tilde{\eta} t}\int_0^t \sum_{i=1}^{N}\tilde{\hbar}_i N(\hat{\rho}_{i,1})\dot{\hat{\rho}}_{i,1} e^{\tilde{\eta} k} dk \tag{71}$$

After rearrangement, one obtains

$$e^{-\tilde{\eta} t}\sum_{i=1}^{N}\tilde{\hbar}_i \int_0^t N(\hat{\rho}_{i,1})\dot{\hat{\rho}}_{i,1} e^{\tilde{\eta} k} dk \ge -C \tag{72}$$

where $C = V(0) + \frac{\tilde{b}}{\tilde{\eta}}$. Hence each integral $\int_0^t N(\hat{\rho}_{i,1})\dot{\hat{\rho}}_{i,1} dk$ is bounded from below.

Next, we prove that $\hat{\rho}_{i,1}$ is bounded by contradiction. Assume that $\hat{\rho}_{i,1}$ is unbounded, then there exists a sequence $t_k \to \infty$ such that $\left| \hat{\rho}_{i,1}(t_k) \right| \to \infty$. By the property of a Nussbaum function (Definition 1),

$$\int_0^{t_k} N(\hat{\rho}_{i,1})\dot{\hat{\rho}}_{i,1} dk = \int_{\hat{\rho}_{i,1}(0)}^{\hat{\rho}_{i,1}(t_k)} N(s) ds \to -\infty \tag{73}$$

which contradicts the lower bound in (72). Accordingly, every $\hat{\rho}_{i,1}(t)$ is bounded, and consequently $\int_0^t N(\hat{\rho}_{i,1})\dot{\hat{\rho}}_{i,1} dk$ is bounded as well. Therefore, with $\Upsilon(t) = \hat{\rho}_{i,1}(t)$, inequality (67) meets the requirements of Lemma 1.

All signals in the above inequality are bounded under Lemma 1. Without a doubt, $y_{t,j}$ is bounded due to Assumption 1. So $y_i = x_{i,1}$ is also bounded. From the definition of $\hat{\theta}_{i,m}$, one concludes that $\hat{\theta}_{i,m}$ is bounded. Obviously, all signals of MASs (1) are bounded.

Under the conditions of Theorem 1, the containment error $z_{i,1}$ satisfies the following uniform ultimate boundedness (UUB) estimate:

$$\limsup_{t \to \infty} \left| z_{i,1}(t) \right| \le \left(\frac{r_i \overline{b}}{\eta} \right)^{1/r_i} \tag{74}$$

where $r_i = p_i - p_{i,1} + 2$, $\overline{b} = b + sup_t \left| \mathrm{W}(t) \right|$ is a finite constant because W is bounded.

From the proof of Theorem 1 we have $\dot{V} \le -\eta V + b + \mathrm{W}$. It has been proven that W is bounded, there exists a constant $\overline{b} > 0$ such that $\dot{V} \le -\eta V + \overline{b}$. Solving this differential inequality yields

$$V(t) \le V(0)e^{-\eta t} + \frac{\overline{b}}{\eta}\left(1 - e^{-\eta t} \right) \tag{75}$$

Because the Lyapunov function V contains the term $\frac{z_{i,1}^{r_i}}{r_i}$, we have $\frac{z_{i,1}^{r_i}}{r_i} \le V(t)$, hence

$$\limsup_{t\to\infty} |z_{i,1}(t)| \le \left(\frac{r_i \bar{b}}{\eta}\right)^{1/r_i} \tag{76}$$

Since $\eta_i = \min\left\{\kappa_{i,k}, \xi_{i,n} - \frac{1}{2}, \omega_{i,k}\right\}$, so increasing $\kappa_{i,k}$ or $\omega_{i,k}$ will increase η and thus reduce the steady-state error bound. Moreover, because b_i contains terms such as $\frac{\omega_{i,k}}{c_{i,k}}$, $\lambda_{i,k}^{-\frac{p_i+1}{p_{i,k}}}$, and $\varepsilon_{i,k}^{\frac{p_i+1}{p_{i,k}}}$, increasing $c_{i,k}$ and $\lambda_{i,k}$ while decreasing $\varepsilon_{i,k}$ will reduce b_i, thereby reducing the steady-state error bound and improving the steady-state performance.

5. Simulation Results

A numerical example is performed to verify the validity of developed control protocol. As shown in Fig. 1, one considers nonlinear p-normal MASs which are consisted of three followers (1, 2, 3) and two leaders (L_1, L_2). The leader adjacency matrix, the adjacency matrix T_A, and the Laplacian matrix T_L are respectively as

$$\mathrm{T}_D = \mathrm{diag}(4,2,2,0,0),$$

$$\mathrm{T}_A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathrm{T}_L = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ 0 & 2 & -1 & -1 & 0 \\ -1 & 0 & 2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Consider the following nonlinear p-normal MASs as

$$\begin{cases} \dot{x}_{i,1} = \tilde{n}_{i,1} x_{i,2} + \psi_{i,1}^T(t)\Xi_{i,1} \\ \dot{x}_{i,2} = \tilde{n}_{i,2} u_i^3 + \psi_{i,2}^T(t)\Xi_{i,2} + d_{i,2}(t) \\ y_i = x_{i,1}, (i = 1,2,3) \end{cases} \tag{76}$$

where $\tilde{n}_{1,1} = 0.2 + 0.01\sin^2(x_{1,1})$, $\tilde{n}_{2,1} = 0.3 + 0.01\sin^2(x_{2,1})$, $\tilde{n}_{3,1} = 0.2 + 0.01\sin(x_{3,1})$, $\tilde{n}_{i,2} = 0.2 + 0.1\sin(x_{i,2})$, $\psi_{i,1}(t) = 0.01\cos(0.1t+1)\cos(0.1t+0.1)$, $\psi_{i,2}(t) = 0.1\cos(0.1t+0.1)$, $\Xi_{i,1} = 0.1x_{i,1} + 0.1\cos(x_{i,1})e^{-(x_{i,1}-3)}$, $\Xi_{i,2} = 10x_{i,2}\sin(x_{i,2})$ and $d_{i,2} = 1.5 + 1.5x_{i,1}x_{i,2}\cos(t)$, for $i = 1,2,3$.

Two leaders are chosen as $y_{l,1} = 0.5\sin(0.999t) - 0.05$ and $y_{l,2} = 0.5\sin(0.999t) - 0.24$. The design parameters are set as: $c_{i,1} = 5$, $c_{i,2} = 1$, $\kappa_{1,1} = 0.1$, $\kappa_{2,1} = \kappa_{3,1} = 1$, $\kappa_{i,2} = 8$, $\omega_{i,1} = 1$, $\omega_{i,2} = 0.2$, $r_{i,1} = 10$, where $i = 1,2,3$.

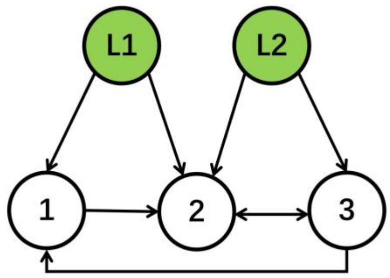

Figure 1. Directed communication graph.

The simulation results are presented in Figs. 2-6. Under the proposed containment control method, it is clearly observed that three followers are driven into a convex hull crossed by two leaders in Fig. 2. The curves of $z_{i,1}$ and $\hat{\theta}_{i,1}$ are described in Figs. 3 and 4 separately. The outlines of the controller u_i are represented in Fig. 5. Fig. 6 depicts the disturbance observer errors. In Fig. 7, it is clearly seen that three followers cannot be driven into the convex hull crossed by two leaders before 6.5 seconds without designing the disturbance observer. According to the simulation results, it is consistent with the conclusion in Theorem 1.

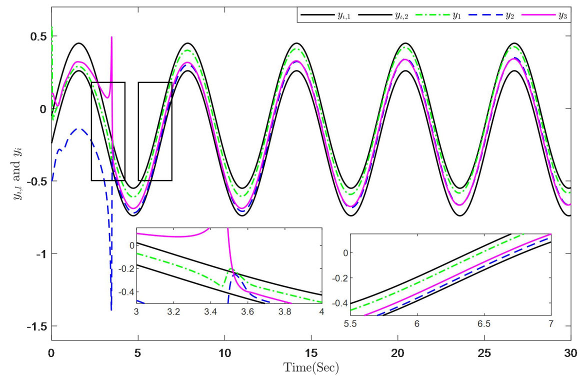

Figure 2. State trajectories of leaders $y_{l,1}$, $y_{l,2}$ and followers y_1, y_2, y_3.

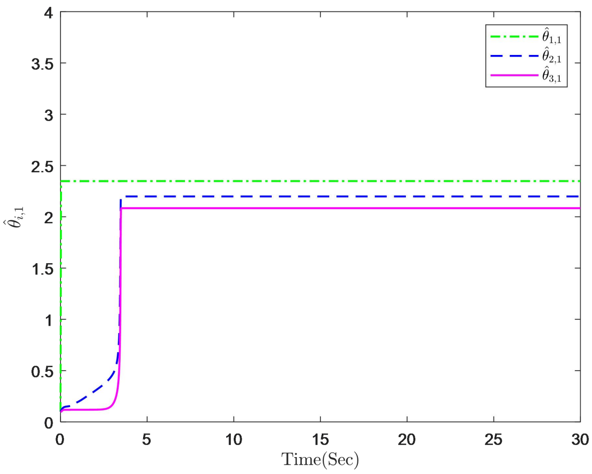

Figure 3. Curves of adaptive laws.

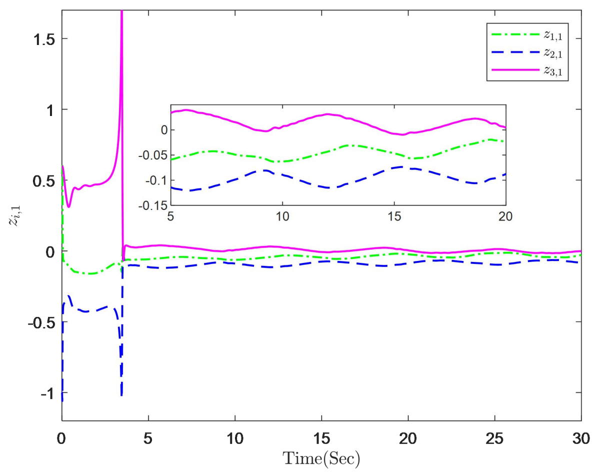

Figure 4. Trajectories of containment errors.

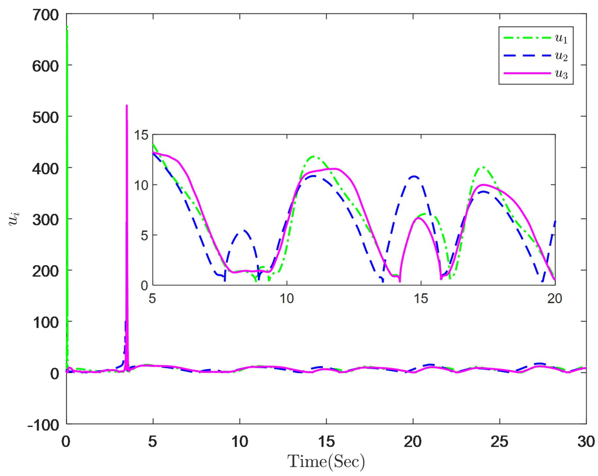

Figure 5. Control signals u_1, u_2 and u_3.

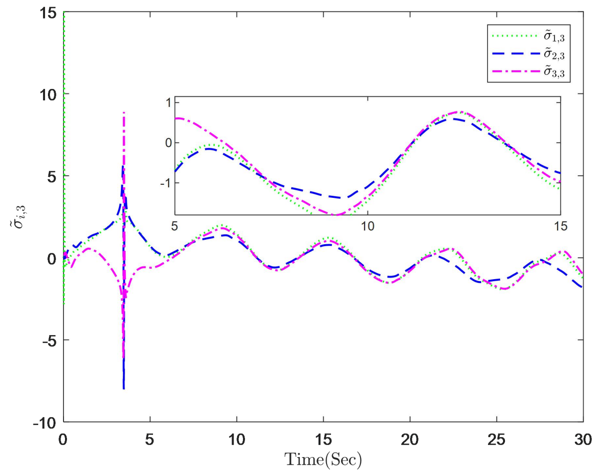

Figure 6. Disturbance observer errors $\tilde{\sigma}_{1,3}$, $\tilde{\sigma}_{2,3}$ and $\tilde{\sigma}_{3,3}$.

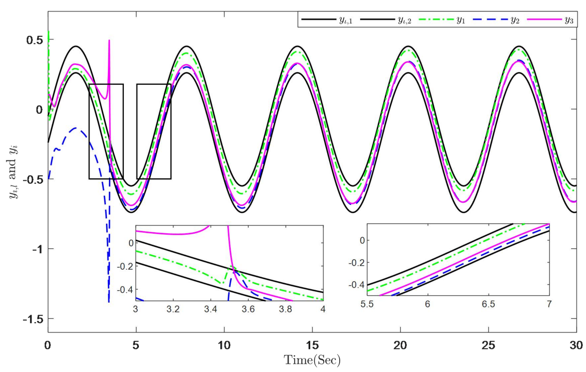

Figure 7. State trajectories of leaders $y_{\iota,1}$, $y_{\iota,2}$ and followers y_1, y_2, y_3 without disturbance observer.

Remark 5. The disturbance observer is established to cope with the disturbances in nonlinear p-normal MASs. According to the intermediate information of nonlinear p-normal MASs, the disturbance can be effectively estimated and addressed by the designed control strategy. Compared with Fig. 7, due to the design of the observer and obtaining the observer gain value $\xi_{i,n}=1$, the followers in Fig. 2 converge to a convex hull established by the leaders.

6. Conclusions

This paper has presented the adaptive containment control protocol for nonlinear p-normal MASs with sensor faults, disturbances and unknown time-varying parameters. The problem of power exponential terms being positive odd in nonlinear p-normal MASs has been solved by using an approach called adding a power integrator technique. Furthermore, sensor faults have been resolved via using

Nussbaum gain technology. The FLSs have been utilized to handle the negative impacts of nonlinear functions and unknown time-varying parameters. A nonlinear disturbance observer has been constructed to estimate unknown composite disturbances. The adaptive optimal control problem for nonlinear p-normal MASs is a meaningful topic for future work.

Author Contributions: Conceptualization, Zhirui Li and Liang Cao; validation, Meng Zhao; writing—original draft preparation, Zhirui Li; writing—review and editing, Lu Liu and Liang Cao; visualization, Xiaomeng Li and Wenbin Xiao; funding acquisition, Liang Cao, Meng Zhao, Xiaomeng Li and Wenbin Xiao. All authors have read and agreed to the published version of the manuscript.

Funding: This work was partially supported by the National Natural Science Foundation of China (62473059, 62303179, 62303125), and the Natural Science Foundation of Liaoning Province (2024MS183, 2024BS235).

Ethical Approval: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data that support the findings of this paper are available from the corresponding author upon reasonable request.

Conflicts of Interest: The authors declare that there are no potential conflicts of interest regarding the publication of this work. And there are no financial and personal relationships with other people or organizations that can inappropriately influence our work.

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